B. Sc. Vth Semester Physics-XV

Ch. 3. Wave Particle Duality

1. Introduction:

The phenomenon of interference and diffraction which are results of interaction of light with several other phenomena such as reflection, refraction and polarisation are explained on the basis of wave theory. These phenomena prove that light possesses wave nature. On the other hand, the phenomena of black body radiation, photoelectric effect and Compton effect, which are the result of interaction of radiation with matter cannot be explained by wave theory. For explaining them, we consider radiant energy as a stream of small bundles of energy $h\nu$ which are called quanta or photons. It proves that light possesses particle nature. Thus in certain events radiation shows particle like character, when a light beam is diffracted by grating it shows wave like character but when it causes an emission of photoelectrons from a surface, it shows particle like character. This shows that radiation possesses dual character behaving as a wave in one situation and as a particle in other situation.

2. De Broglie's Hypothesis of Matter Waves:

Louis de Broglie in 1924 put forward theory of matter that matter also shows dual nature. His hypothesis of dual nature of matter based on two assumptions:-

i) The whole energy in universe is in form of electromagnetic radiations and matter.
ii) The nature is symmetrical i.e. matter and energy must be symmetrical that means radiations having dual nature, matter must also posses dual nature i.e. wave as well as particle.

Matter considered to be made up of discrete particles such as atoms and molecules may also behave like wave under proper conditions called as matter waves. The wave nature radiation has been well established by the phenomenon of diffraction and interference. Electrons, light rays or X-rays have also been shown to exhibit the diffraction phenomenon. This leads to view that electrons like light may also have wave properties associated with them. It means an electron has dual nature i.e. particle and wave.

Consider a moving particle; it has wave properties associated with it. The wavelength λ associated with momentum p then we have,

$$\lambda = \frac{h}{p} = \frac{\mathrm{h}}{\mathrm{mv}}$$

Where m is mass of particle moving with velocity v.

We know that the relativistic mass is,

$$m = \frac{m_0}{\sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}}}$$

The waves associated with moving particles are known as matter waves.

Proof of De Broglie Wavelength:

By using plank's theory of radiation, the energy of photon is $E = h\nu$ where h is plank's constant and ν is frequency of radiation.

But frequency of radiation ν = c/λ where c is velocity of light and λ be wavelength.

$$\therefore E = \frac{hc}{\lambda} \quad - - - - - - - - (1)$$

If a particle of mass m is converted into energy, the equivalent energy is given by Einstein mass-energy relation as,

$$E = mc^2 \quad - - - - - - - - (2)$$

Equating $eq^n(1)$ and $eq^n(2)$,

$$\frac{hc}{\lambda} = mc^2 \quad \therefore \lambda = \frac{h}{mc} \quad ------(3)$$

But mc = p be momentum associated with quanta.

$$\therefore \lambda = \frac{h}{p} \quad --------(4)$$

De Broglie suggested that $eq^n(4)$ is a completely general one that applies to material particle as well as photons.

The momentum of particle of mass *m* and velocity v is p = mv

Therefore De Broglie Wavelength is,

$$\lambda = \frac{h}{m\mathrm{v}}$$

Let E_k is kinetic energy of material particle then

$$E_k = \frac{1}{2}m\mathrm{v}^2 = \frac{1}{2}\frac{m^2\mathrm{v}^2}{m} = \frac{p^2}{2m}$$

$$\therefore p^2 = 2mE_k$$

$$\therefore p = \sqrt{2mE_k}$$

De Broglie wavelength in terms of kinetic energy is written as

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} \quad ------(5)$$

If material particle are in thermal equilibrium at associated temperature T then,

$$E_k = \frac{3}{2}KT$$

Where K is Boltzmann constant = $1.38\times10^{-23} J/^0K$.

∴ De Broglie wavelength is,

$$\lambda = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{3mKT}} \text{ -------(6)}$$

If a charged particle carrying charge q is associated through a potential difference V then kinetic energy is $E_k = qV$.

∴ De Broglie wavelength is,

$$\lambda = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{2mqV}} \text{ -------(7)}$$

3. Phase Velocity or (Wave velocity) of De Broglie Wave:

A wave is a disturbance from equilibrium condition that travels with time from the region of space to another. The particle begins to vibrate about mean position. There is a progressive change of phase from one particle to the next.

The phase relationship of these particles is called as wave and the velocity with which planes of constant phase propagates through medium is known as wave velocity or phase velocity.

Consider the equation of plane progressive wave,

$$y = a\,sin\,(wt - kx)$$

Where $w = \frac{2\pi}{T} = 2\pi\nu$ is angular frequency and

$k = \frac{2\pi}{\lambda}$ is propogation constant

The term *(wt-kx)* represents the phase of wave motion. Hence, planes of constant phase are written as,

$$wt\text{-}kx = constant.$$

Differentiate with respect to t, we get

$$w - k\frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = \frac{w}{k} = V_p$$

Where V_p is called as phase velocity or wave velocity.

$$\therefore \text{ phase velocity } V_p = \frac{w}{k} ------(1)$$

$$\text{phase velocity } V_p = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda ------(2)$$

De Broglie wavelength of material particle is given by, $\lambda = \frac{h}{mV}$

Let E is energy of wave and $E = h\nu$ then its frequency is $\nu = E/h$

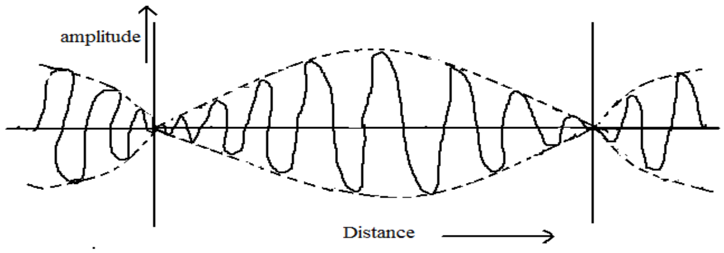
$$\therefore \text{ phase velocity } V_p = \frac{E}{h}\frac{h}{mV} = \frac{E}{mV}$$

From Einstein mass energy relation $E = mc^2$

$$\therefore \text{ phase velocity } V_p = \frac{mc^2}{mV} = \frac{c^2}{V}$$

Since c >> V i.e. particle velocity is always less than the velocity of light. Therefore, the De Broglie wave velocity V_p must be greater than velocity of light c.

4. Group Velocity:-



Consider two waves that have the same amplitude A but differ by angular frequencies and propagation constant as shown in figure.

Le w and Δw be angular frequencies; k and Δk be propagation constant.

$\therefore$ Equations of waves are,

$$y_1 = A\cos(wt - kx) \quad and \qquad y_2 = A\cos[(w + \Delta w)t - (k + \Delta k)x]$$

The superposition of the two waves will give a single wave packet or group velocity. Let us find the velocity V_g with which the wave group travels.

The resultant displacement at any time t and any position x is given by,

$$y = y_1 + y_2$$

$$y = A\cos(wt - kx) + A\cos[(w + \Delta w)t - (k + \Delta k)x]$$

By using relation,

$$cosC + cosD = 2\cos\left(\frac{C + D}{2}\right).\cos\left(\frac{C - D}{2}\right)$$

$$\therefore y = 2A\cos\left[\frac{wt - kx + (w + \Delta w)t - (k + \Delta k)x}{2}\right] \times$$

$$\cos\left[\frac{wt - kx - (w + \Delta w)t + (k + \Delta k)x}{2}\right]$$

$$= 2A\,cos\left[\frac{wt - kx + wt + \Delta wt - kx - \Delta kx}{2}\right] \times$$

$$cos\left[\frac{wt - kx - wt - \Delta wt + kx + \Delta kx}{2}\right]$$

$$= 2A\cos\left[\frac{2wt - 2kx + \Delta wt - \Delta kx}{2}\right] \times \cos\left[\frac{-\Delta wt + \Delta kx}{2}\right]$$

$$y = 2A\cos\left[\frac{(2w + \Delta w)t - (2k + \Delta k)x}{2}\right] \times \cos\left[\frac{\Delta wt - \Delta kx}{2}\right]$$

Since Δw and Δk are very small as compared with w and k respectively.

$\therefore\ 2w + \Delta w \approx 2w$ *and* $2k + \Delta k \approx 2k$

$$\therefore y = 2A\cos\left[\frac{2wt - 2kx}{2}\right].\cos\left[\frac{\Delta wt - \Delta kx}{2}\right]$$

$$y = 2A\ \cos\left[\frac{\Delta w}{2}t - \frac{\Delta k}{2}x\right]\cos[wt - kx] ------(1)$$

This is analytical expression for resultant wave due to superposition of two waves. The quantity $2A\cos[\frac{\Delta w}{2}t - \frac{\Delta k}{2}x]$ is considered to be an amplitude of wave which varies with x and t. This variation of amplitude is called modulation of wave.

Hence $eq^n(1)$ represent a wave of angular frequency w and wave number k that is superimposed upon it a modulation of angular frequency ½ Δw.

The velocity V_g of wave group is,

$$V_g = \frac{\Delta w}{\Delta k}$$

When w and k have continuous, the group velocity is

$$V_g = \frac{dw}{dk}$$

This is expression of group velocity.

5. Relation between group velocity and particle velocity:

According to De Broglie hypothesis, a particle moving with a velocity v is supposed to consist of a group of waves.

The group velocity is given by,

$$V_g = \frac{dw}{dk}$$

The angular frequency of De Broglie waves associated with a particle of rest mass m_0 moving with velocity v is given by,

$$w = 2\pi\nu$$

But $E = h\nu = mc^2$

$$\therefore \nu = \frac{mc^2}{h}$$

$$\therefore \qquad w = \frac{2\pi mc^2}{h}$$

The relativistic mass of particle moving with velocity v is

$$m = \frac{m_0}{\sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}}}$$

$$\therefore \quad w = \frac{2\pi m_0 c^2}{h\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} \quad --------------(1)$$

Differentiate eq^n(1) with respect to v, we get

$$\frac{dw}{d\mathrm{v}} = \frac{2\pi m_0 c^2}{h} \frac{d}{d\mathrm{v}} \left[1 - \frac{\mathrm{v}^2}{c^2}\right]^{-1/2}$$

$$= \frac{2\pi m_0 c^2}{h} \left[-\frac{1}{2}\left(1 - \frac{\mathrm{v}^2}{c^2}\right)^{-3/2} . -\frac{2\mathrm{v}}{c^2}\right]$$

$$\frac{dw}{d\mathrm{v}} = \frac{2\pi m_0 \mathrm{v}}{h\left(1 - \frac{\mathrm{v}^2}{c^2}\right)^{3/2}} \quad -------------(2)$$

We know that propagation constant, $k = {}^{2\pi}/_{\lambda}$ $\quad But\ \lambda = {}^{h}/_{m\mathrm{v}}$

$$k = \frac{2\pi m\mathrm{v}}{h} = \frac{2\pi m_0 \mathrm{v}}{h\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}$$

Differentiate with respect to v, we get

$$\frac{dk}{d\mathrm{v}} = \frac{2\pi m_0}{h} \frac{d}{d\mathrm{v}} \left[\frac{\mathrm{v}}{\sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}}}\right]$$

$$= \frac{2\pi m_0}{h} \left[\frac{\sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}}\ .1 - \mathrm{v}.\frac{1}{2\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} . \frac{-2\mathrm{v}}{c^2}}{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}}\right]$$

$$= \frac{2\pi m_0}{h} \left[\frac{\sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}} + \frac{1}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} . \frac{\mathrm{v}^2}{c^2}}{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}}\right] = \frac{2\pi m_0}{h} \left[\frac{1 - \frac{\mathrm{v}^2}{c^2} + \frac{\mathrm{v}^2}{c^2}}{\left(1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}\right)^{3/2}}\right]$$

$$\frac{dk}{d\mathrm{v}} = \frac{2\pi m_0}{h\left(1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}\right)^{3/2}} \text{ - - - - - - - - } (3)$$

The group velocity of De Broglie wave associated with particle is given by

$$V_g = \frac{dw}{dk} = \frac{2\pi m_0 \mathrm{v}}{h\left(1 - \frac{\mathrm{v}^2}{c^2}\right)^{3/2}} \times \frac{h\left(1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}\right)^{3/2}}{2\pi m_0}$$

$$V_g = \mathrm{v}$$

This is relation between group velocity and particle velocity. This shows that De Broglie wave group associated with a moving particle travels with same velocity as that of particle.

6. Relation between group velocity (V_g) and phase velocity (V_p):

We know that, phase velocity $V_p = w/k$ and group velocity $V_g = dw/dk$

The wave number,

$$k = \frac{2\pi}{\lambda} \qquad \therefore \frac{dk}{d\lambda} = \frac{-2\pi}{\lambda^2} \text{ - - - - - - - } (1)$$

And angular frequency,

$$w = 2\pi\nu = \frac{2\pi V_p}{\lambda}$$

$$\therefore \frac{dw}{d\lambda} = 2\pi \frac{d}{d\lambda}\left[\frac{V_p}{\lambda}\right] = 2\pi\left[\frac{-V_p}{\lambda^2} + \frac{1}{\lambda}.\frac{dV_p}{d\lambda}\right]$$

$$\therefore \frac{dw}{d\lambda} = -\frac{2\pi}{\lambda^2}\left[V_p - \lambda\frac{dV_p}{d\lambda}\right] \text{ - - - - - - - } (2)$$

Dividing $eq^n(2)$ by $eq^n(1)$ we get

$$\frac{dw}{dk} = \frac{dw/_{d\lambda}}{dk/_{d\lambda}} = \frac{-\frac{2\pi}{\lambda^2}\left[V_p - \lambda\frac{dV_p}{d\lambda}\right]}{\frac{-2\pi}{\lambda^2}}$$

$$\frac{dw}{d\lambda} = V_p - \lambda.\frac{dV_p}{d\lambda}$$

$$V_g = V_p - \lambda.\frac{dV_p}{d\lambda} - - - - - - - - (3)$$

This is relation between group velocity and phase velocity.

From $eq^n(3)$ following two cases arises

Case I: For dispersive medium

Phase velocity V_p is function of wavelength λ i.e. $V_p= f(\lambda)$. Usually $dV_p/d\lambda$ is positive. Therefore, $V_g < V_p$. This is case of De Broglie waves.

Case II: For non- dispersive medium

Phase velocity V_p is not function of wavelength λ i.e. $V_p \neq f(\lambda)$.

Then $dV_p/_{d\lambda} = 0 \quad \therefore V_g = V_p$

This result is true for electromagnetic waves in vacuum.

7. Davisson and Germer's experiment:

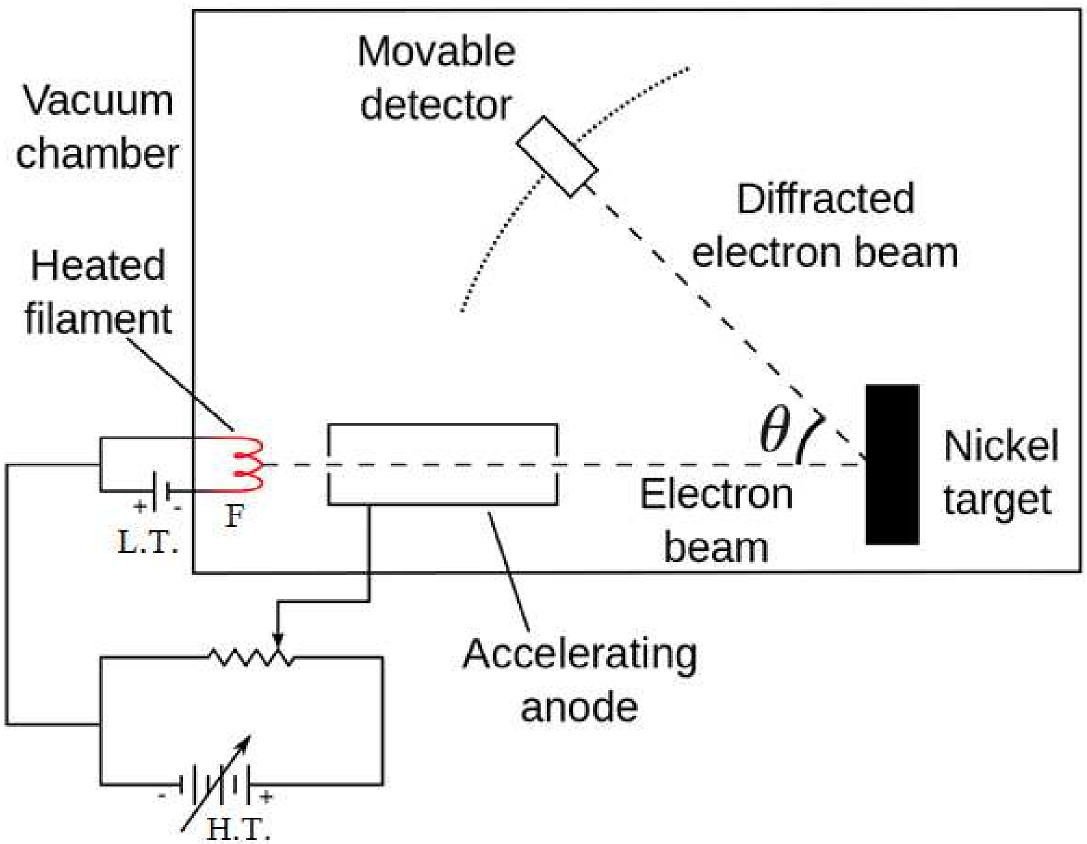


Figure 1. Experimental set up of Davisson Germer's experiment

The experimental arrangement to study matter waves as shown in Fig 1. The whole set-up is enclosed in an evacuated chamber. It consists of an electron gun which produce electron by heating a filament (F) by a low tension battery (L.T.). The electrons from gun are accelerated through vacuum to a desired velocity by applying a suitable accelerating potential to accelerating anode and are collimated into focused beam. This electron beam falls on a large single crystal of nickel. The electrons are scattered in all directions by the atoms in the crystal. The detector can be moved to any angle ϕ relative to incident beam. The energy of the electrons in the primary beam, the angle at which they reach the target and position of detector could all be varied.

Experimental Procedure: Initially very low accelerating potential V is given to an accelerating anode by using high tension battery (H.T.). The beam of electron falls normally on the surface of the crystal. The detector is moved to various positions and the intensity of diffracted beam at each position is noted. The intensity of diffracted beam is plotted against the angle between the incident beam and beam entering the detector. The method of plotting is such that the intensity at any angle is proportional to the distance of the curve at that angle from the point of scattering. The observations are repeated for different accelerating voltages. The intensity of the scattered electrons is not continuous. It shows a maximum and a minimum value corresponding to the maxima and the minima of a diffraction pattern produced by X-rays. It is studied from various angles of scattering and potential difference.

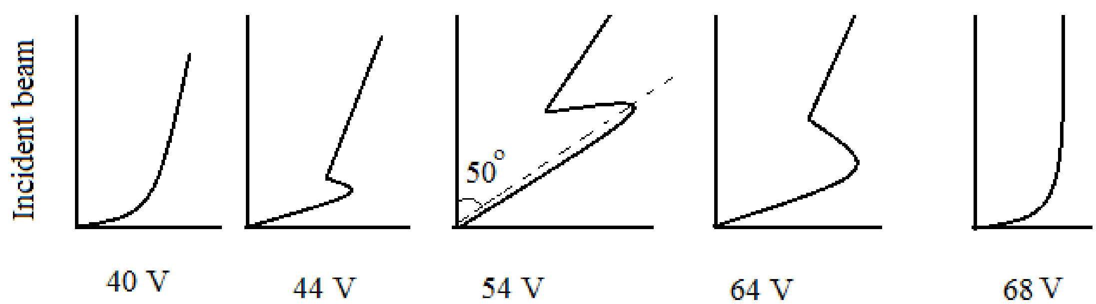


Fig. (2)

Figure 2 shows typical polar graphs of electrons intensity. From graphs we observed that,

i) The graph remains fairly smooth till the accelerating voltage becomes 44V and then bump appears on the curve.
ii) As the accelerating voltage is increased, the length of bump increases.
iii) The bump becomes most prominent in the curve for 54V at angle $\phi = 50^{o}$.
iv) As the accelerating voltage is further increased the bump decreases in length and finally disappears at 68V.

For nickel, the spacing of the atomic planes which can be measured by X-ray diffraction is d =0.91nm. There is an intense reflection of the beam at angle $\phi = 50^{o}$.

The angle of incidence relative to Bragg planes is,

$$\theta = \frac{180^0 - 50^0}{2} = \frac{130^0}{2} = 65^0$$

The Bragg equation for maxima in the diffraction pattern is,

$$n\lambda = 2dsin\theta$$

For n=1, the De Broglie wavelength λ of diffracted electron is,

$$\lambda = 2 \times 0.091 \times 10^{-9} \times sin65^0$$

$$= 0.182 \times 0.9061 \times 10^{-9}$$

$$\lambda = 0.165 \times 10^{-9} = 0.165\, nm$$

The expected wavelength of electron can be calculated by using De Broglie's formula. The electron kinetic energy is 54eV. The electron wavelength is,

$$\lambda = \frac{h}{\sqrt{2mE_k}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 54 \times 1.6 \times 10^{-19}}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{1572.48 \times 10^{-50}}} = \frac{6.63 \times 10^{-34}}{39.65 \times 10^{-25}}$$

$$\lambda = 0.1672 \times 10^{-9} = 0.1672nm$$

The observed wavelength agrees well with the expected wavelength. The Davisson Germer experiment thus directly verifies De Broglie's hypothesis of the wave nature of moving bodies.

8. Heisenberg's Uncertainty Principle:

Statement: It is impossible to determine precisely and simultaneously the values of both the numbers of a pair of physical variables which describe the motion of an atomic system. Such pairs of variables are called as canonically conjugate variables.

According to this principle, the position and momentum of a particle (say electron) cannot be determined simultaneously to any desired degree of accuracy.

Let Δx as the error in determining its position and Δp be error in determining its momentum at the same instant, these quantities are related as,

$$\Delta x.\Delta p \geq \hbar/2 \qquad \text{Where } \hbar = h/2\pi$$

The product of the two errors is approximately of the order of plank's constant. If Δx is small, Δp will be large and vice versa. It means that if one quantity is measured accurately the other quantity becomes less accurate. Thus any instrument cannot measure the quantities more accurately than predicated by Heisenberg's principle of uncertainty.

The time-energy Uncertainty Principle:

Another form of the uncertainty principle is sometimes useful. Suppose we wish to measure the energy E emitted sometime during the time interval Δt in an atomic process. If the energy is in the form of electromagnetic waves, the limited time available restricts the accuracy with which we can determine the frequency ν of the waves. Since the frequency is the reciprocal of time period, the uncertainty Δν in our frequency measurement is

$$\Delta\nu = 1/(\Delta t) \qquad (1)$$

The corresponding energy uncertainty is

$$\Delta E = h\,\Delta\nu$$

Using equation (1), we have

$$\Delta E = h/(\Delta t)$$

or $$\Delta E . \Delta t = h$$

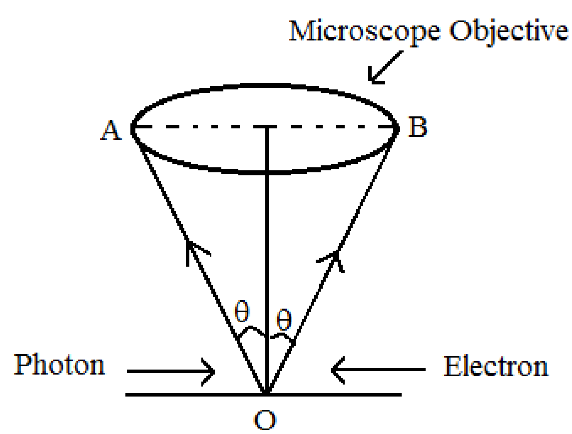
A more realistic calculation changes above equation to

$$\Delta E. \Delta t \geq \hbar/2 \qquad (2)$$

This equation states that the product of the uncertainty ΔE in an energy measurement at any instant and the uncertainty Δt in the time measured at the same instant is equal to or greater than ħ.

The principle of uncertainty can also be expressed in terms of angular momentum and angular displacement i.e. $\Delta\theta . \Delta L_\theta \geq \hbar/_2$.

Proof:



Suppose we try to measure the position and linear momentum of an electron using an imaginary microscope with a very high resolving power as shown in figure.

The resolving power of microscope is given by,

$$\Delta x = \frac{\lambda}{2\, sin\theta}$$

Where Δx is distance between two points which can be just resolved by microscope. This is the range in which electron would be visible when disturbed by photon. Hence Δx is the uncertainty involved in the position measurement of the electron.

However, the incoming photon will interact with the electron. To able to see this electron, the scattered photon should enter the microscope within the angel 2θ.

The momentum imparted by the photon to electron by order of h/λ. The component of this momentum along OA is $-hsin\theta/\lambda$ and along OB is $hsin\theta/\lambda$.

Hence uncertainty in the momentum measurement in the X-direction is,

$$\Delta p_x = \frac{h\, sin\theta}{\lambda} - \left(-\frac{h\, sin\theta}{\lambda}\right) = \frac{2h\, sin\theta}{\lambda}$$

$$\therefore \Delta x. \Delta p_x = \frac{\lambda}{2\, sin\theta} \times \frac{2h\, sin\theta}{\lambda} = h$$

A more approach will shows that $\Delta x. \Delta p \geq {}^{\hbar}/_{2}$, where $\hbar = {}^{h}/_{2\pi}$.

Applications of Uncertainty Principle:-

i) Non-existence of the electrons in the nucleus:

The non-existence of electron in nucleus is proved by comparing the energy needed to an electron to exist in nucleus.

The nucleus of typical atoms have radius 10^{-14}m. If electron is confined to the nucleus the maximum possible uncertainty in its position may be ≈ 2× radius of nucleus.

$\Delta x \approx 2\times10^{-14}$ m.

The uncertainty in the momentum of electron will be,

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{h}{2\pi \Delta x}$$

$$= \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 2 \times 10^{-14}} = \frac{6.63}{12.56} \times 10^{-20}$$

$$= 0.5278 \times 10^{-20} = 5.278 \times 10^{-21} \quad Kg - m/sec$$

The kinetic energy of the electron of mass m is given by,

$$E_k = \frac{p^2}{2m}$$

$$= \frac{(5.278 \times 10^{-21})^2}{2 \times 9.1 \times 10^{-31}} = \frac{27.8256 \times 10^{-42}}{18.2 \times 10^{-31}}$$

$$= 1.5289 \times 10^{-11}\, J$$

$$E_k = \frac{1.5289 \times 10^{-11}}{1.6 \times 10^{-19}}\ eV$$

$$E_k = 0.9556\ \times 10^{8} = 95.56MeV \approx 97MeV$$

Therefore, if the electron exists in the nucleus, it should have energy of the order of 97MeV. However, it is observed that electrons ejected from the nucleus during β –decay have energies of approximately 4MeV, which is quite different from the calculated value of 4 MeV. Second reason that electron cannot exist inside the nucleus is that experimental results show that no electron or particle in the atom possess energy greater than 4 MeV.

ii) Binding energy of an electron in an atom:

Binding energy of an electron in an atom is minimum energy required to free electron from the surface.

Suppose hydrogen atom has radius 5.3×10^{-11}m. Therefore uncertainty in position is $\Delta x = 5.3\times10^{-11}$m

The uncertainty in the momentum of electron will be,

$$\Delta p \geq \frac{\hbar}{\Delta x} \geq \frac{h}{2\pi \Delta x}$$

$$\geq \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 5.3 \times 10^{-11}} \geq \frac{6.63}{33.284} \times 10^{-23}$$

$$\geq 0.199 \times 10^{-23} \quad Kg - m/sec$$

An electron whose momentum is of this order of magnitude behaves like a classical particle and its kinetic energy is

$$E_k \geq \frac{p^2}{2m}$$

$$= \frac{(0.199 \times 10^{-23})^2}{2 \times 9.1 \times 10^{-31}} = \frac{0.0396 \times 10^{-46}}{18.2 \times 10^{-31}}$$

$$= 0.002176 \times 10^{-15}\, J$$

$$E_k = \frac{0.002176 \times 10^{-15}}{1.6 \times 10^{-19}}\ eV$$

$$E_k = 0.00136 \times 10^4 = 13.6eV$$

The kinetic energy of an electron in the lowest energy level of a hydrogen atom is 13.6eV which is binding energy.

Problems:

1. Find the velocity of electron whose De Broglie wavelength is 1.2Å. (h = 6.63×10^{-34} J-S).

   Given: λ = 1.2 Å = 1.2×10^{-10}m, h = 6.63×10^{-34} J-S

   Mass of electron m = 9.1×10^{-31} kg, V = ?

   We know that De Broglie wavelength is given by,

$$\lambda = \frac{h}{mv}$$

   Velocity of electron is,

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.2 \times 10^{-10}}$$

$$v = \frac{6.63}{10.92} \times 10^7 = 0.6071 \times 10^7 = 6.071 \times 10^6 m$$

2. Calculate the De Broglie wavelength of an electron accelerated through 20000V.

   Given : V = 20000V, λ=?

   De Broglie wavelength of an electron having energy E is given by,

$$E = \frac{h}{\sqrt{2mE}}$$

But E = qV where V is potential difference and q be charge on electron.

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 20000}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{58.24 \times 10^{-46}}} = \frac{6.63 \times 10^{-34}}{7.4324 \times 10^{-23}}$$

$$\lambda = 0.892 \times 10^{-11}\, m = 0.0892Å$$

3. A neutron is an uncharged particle of mass 1.67×10^{-27} kg. Calculate the De Broglie wavelength associated with it if its kinetic energy is 0.04eV.

Given: m = 1.67×10^{-27} kg,

E = 0.04eV = $0.04\times1.6\times10^{-19}$ J = 0.064×10^{-19} J

We know that, De Broglie wavelength having in terms of kinetic energy is

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 0.064 \times 10^{-19}}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{0.2138 \times 10^{-46}}} = \frac{6.63 \times 10^{-34}}{0.4624 \times 10^{-23}}$$

$$= 14.34 \times 10^{-11} m = 1.434\text{ Å}$$

4. Calculate De Broglie wavelength of matter waves associated with a ball of mass 0.2 kg moving with a velocity of 20 m/s.

Given: λ=? , m =0.2 kg and v = 20 m/s.

We know that De Broglie wavelength

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{0.2 \times 20}$$

$$= \frac{6.63 \times 10^{-34}}{4} = 1.6575 \times 10^{-34} m$$

5. If the life time of an electronic excited state is 1×10^{-9} s, what is the uncertainty in the energy of the state?

   Given: Δt = 1×10^{-9} s, ΔE =?

   According to uncertainty principle,

   $\Delta E . \Delta t = \hbar/_2$

$$\therefore \Delta E = \frac{\hbar}{\Delta t \times 2} = \frac{h}{\Delta t \times 4\pi} = \frac{6.63 \times 10^{-34}}{1 \times 10^{-9} \times 4 \times 3.14}$$

$$\therefore \Delta E = \frac{6.63 \times 10^{-25}}{12.56}$$

$$\therefore \Delta E = 0.5278 \times 10^{-25} J$$

Multiple choice questions:

1. According to De Broglie hypothesis electron has dual nature as

   a. Wave b. particle **c. wave as well as particle** d. none of these

2. The De Broglie wavelength associated with the particle of mass m moving velocity v is given by

   a. **h/mv** b. m/hv c. mv/h d. v/mh

3. The De Broglie wavelength of a particle is related to its kinetic energy E as

   a. λ∝E b. $\lambda \propto \sqrt{E}$ c. λ∝1/E **d.** $\boldsymbol{\lambda \propto 1/\sqrt{E}}$

4. The wavelength of de Broglie waves associated by a particle of mass 10 kg moving with velocity 5m/s is

   a. **1.33×10^{-35} m** b. 1.33×10^{-34} m c. 5.33×10^{-35} m d. 1.33×10^{-33} m

5. The relation between De Broglie phase velocity Vp and velocity of light C is

a. $V_p = C$ **b. $V_p > C$** c. $V_p < C$ d. None of these

6. Which of the following relation of De Broglie wavelength of associated particle is correct?

   a. $\lambda = h/\sqrt{2mE}$ b. $\lambda = h/\sqrt{3mKT}$ c. $\lambda = h/\sqrt{2mqV}$ **d. all of these**

7. The relation between group velocity V_g and particle velocity V is

   a. **$V_g = V$** b. $V_g > V$ c. $V_g < V$ d $V_g = V/C$

8. The relation between group velocity V_g and phase velocity V_p is

   a. $V_g = V_p + \lambda.\frac{dV_p}{d\lambda}$ **b. $\boldsymbol{V_g = V_p - \lambda.\frac{dV_p}{d\lambda}}$**

   c. $V_p = V_g - \lambda.\frac{dV_p}{d\lambda}$ **d.** $V_g = V_p$

9. In Davisson and Germer experiment maximum intensity was observed for

   a. 54°, 50V b. 64°, 40V **c. 50°, 54V** d. 44°, 50V

10. In Davisson and Germer experiment, a crystal used to diffract the electron beam is-

    a. Sodium chloride **b. nickel** c. silver d. copper

11. The main aim of Davisson and Germer experiment was to verify

    a. the wave nature of light b. the quantum nature of light

    c. wave nature of electron d. negative charge on electron

12. In Davisson and Germer experiment, a detector can be rotated on a circular scale, the intensity of diffracted beam is

    a. remains constant b. increases continuously

    c. decreases continuously **d. increases becomes maximum and**

decreases

13.An electron is having a kinetic energy of 50eV. Its de Broglie wavelength is

a. 1.732Å b. 2.5 Å c. 4.414 Å d. 6.5 Å

14. The Heisenberg uncertainty principle is

a. $\boldsymbol{\Delta p.\Delta x \geq \hbar/2}$ b. $\Delta p.\Delta x \leq \hbar/2$

c. $\Delta p.\Delta x = \hbar$ d. $\Delta p.\Delta x = \hbar/2$

15. Which of the following Heisenberg uncertainty principle is correct?

a. $\Delta p.\Delta x \geq \hbar/2$ b. $\Delta E.\Delta t \geq \hbar/2$

c. $\Delta L_\theta.\Delta\theta \geq \hbar/2$ **d. All of these**

16. The nucleus of typical atoms have radius

a. 10^{-14}m b. 10^{-10}m c. 10^{-12}m d. 10^{-9}m

17. The duration of radar pulse is 10^{-6} s. The uncertainty in its energy will be—

a. 0 b. 1.05×10^{-35} J **c. 1.05×10^{-28} J** d. 1.05×10^{-21} J

18. The uncertainty in position is 5.3×10^{-11}m then uncertainty in momentum is

a. $0.399 \times 10^{-23} Kg - m/s$ b. $4.99 \times 10^{-23} Kg - m/s$

c. $0.199 \times 10^{-15} Kg - m/s$ **d. $\boldsymbol{0.199 \times 10^{-23} Kg - m/s}$**

19. According to Heisenberg uncertainty principle, the position and momentum of electron ---

a. can be determined simultaneously b. cannot be determined simultaneously

c. sometimes determined d. none of these

20. Uncertainty principle applies to—

a. macroscopic particle **b. microscopic particle**

c. gases d. none of these